CS 70 离散数学 和 概率论
DIS 1A
春季 2026 Sinclair, Song
归纳 Intro
Naturalnumbersstartat0,andthereisalwaysanextone.
Forpredicatesonnaturalnumbers theprinciple
Note3
ofinductionis: n∈N,P(n)≡P(0)∧ n,P(n) = P(n+1).
即, to 证明 P(n) 对于 natural numbers one proves P(0), the 基础情况,
和 n,P(n) = P(n+1),
theinductionstep.
Intheinductionstep,theassumptionthatP(n)istrueiscalledtheinductionhypothesis
whichistypicallyusedtoarguethatP(n+1)istrue.
AnexampleisthestatementP(n)=Σ n i= n(n+1) .
Thebasecase,P(0),istheobservationthatΣ 0 i=0.
i=0 2 i=0
In the 归纳 step, the 归纳 假设, P(n), is Σ n i= (n)(n+1) . The 归纳
step proceeds as
i=0 2
follows:
n+1 n (n)(n+1) (n+1)(n+2)
Σi= Σi+n+1= +n+1= .
2 2
i=0 i=0

Thefirstequalityfollowsfromthedefinitionofthenotation,Σ,thesecondsubstitutestheinductionhypoth-
esisandthelastisalgebra.
AndwhatisprovenisP(n+1),whichisthatΣ n+1i= (n+1)(n+2) .
i=0 2

Anotherandequivalentviewofthenaturalnumbersarethatthereare thenumbers0tonandthenthereis
n+1. Thestronginductionprincipleisthat
n∈N,P(n)≡P(0)∧ n,((k≤n)P(k)) = P(n+1).
Here the 归纳 假设 is that P(k) is 真 对所有 values k≤n. To 证明 that
every natural number

n≥2canbewrittenasaproductofprimes,wetakethebasecaseasP(2)whichcanbewrittenas2,其中
isaproductofaprime.
Andforanyn,ifitisprime,itcanbewrittenasitself,otherwisen=abandbythe

inductivehypothesesP(a)andP(b)isthateachcanbewrittenasaproductofprimes.
因此, wecanwrite
nastheproductoftheprimesinbothaandb.
Noteherethatthebasecasestartsat2,whichillustratesthat
onechosethebasecaseasisrelevanttothestatementbeingproven.
Strengthening the 归纳 假设 is a technique that proves a
stronger 定理. 对于 例子, the
notes 考虑 the 定理 "The sum of the first n odd numbers is a
perfect square." In fact, the notes
inductively 证明 the stronger 定理 "The sum of the first n odd
numbers is n2." Here, the stronger
inductive 假设 allows the 归纳 step to proceed easily. 笔记 that
in strong 归纳, we 假设
more cases are 真 in the inductive 假设, whereas strengthening
the inductive 假设 proves a
strongerclaimentirely.

1 Natural 归纳 on Inequality
Provethatifn∈Nandx>0,那么(1+x)n≥1+nx.

Note3
2 Make It Stronger
Supposethatthesequencea ,a ,... isdefinedbya =1anda =3a2
forn≥1. Wewanttoprovethat
Note3 1 2 1 n+1 n
a
≤3(2n)
n
foreverypositiveintegern.
(a) Supposethatwewanttoprovethisstatementusinginduction.
Canweletourinductivehypothesisbe
simplya
≤3(2n)?

Attemptaninductionproofwiththishypothesistoshowwhythisdoesnotwork.
n
(b)
Instead,令’strytoinsteadproveastrongerstatementinstead,oftheforma
≤3(2n ??).
Whatshould
n
replace the 问题 marks 所以 that the 归纳步骤 works out? Try some
examples 和 see what
works.

(c)
Usingyourstrongerinductivehypothesisfrompart(b),carryouttheinductionproof.
(d) Whydoesthehypothesisinpart(b)implytheoverallclaim?
3 Binary Numbers
证明 that every 正的 整数 n can be written in binary. 换句话说, 证明
that 对于 any 正的
Note3
integern,wecanwrite
n=c ·2k+c ·2k 1+···+c ·21+c ·20,
k k 1 1 0
forsomek∈Nandc ∈{0,1}foralli≤k.
i

Asanexample,thenumber13canbewritteninbinaryas1101because13=1·23+1·22+0·21+1·20.

4 Fibonacci 对于 Home
Recall,theFibonaccinumbers,definedrecursivelyas
Note3
F =1,F =1,andF =F +F .
1 2 n n 2 n 1
ProvethateverythirdFibonaccinumberiseven. Forexample,F
=2isevenandF =8iseven.
3 6